JOURNAL OF AIRCRAFT
Vol. 32, No. 1, January–February 1995

Stability Derivatives of a Flapped Plate in Unsteady Ground Effect

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The stability derivatives of a flapped plate moving in ground proximity are evaluated using an unsteady ground effect model. The model, which has been developed previously, was extended to account for the flap. The results show that ground proximity has a substantial effect on the derivatives of the aerodynamic coefficients. The derivatives obtained by the present unsteady model disagree with those obtained by the customary steady ground effect model, especially very close to the ground. The effect of the flight-path angle (in addition to the pitch angle or angle of attack) on aerodynamic coefficients is found to be substantial and, therefore, their derivatives with respect to it are very important. Because of its own nature, the steady approach fails to account for the flight-path angle and, therefore, it cannot provide the derivatives with respect to this angle near ground.

Nomenclature

A_{ij} = normal component of velocity induced at control point i by a unit circulation at node j
C_l = lift coefficient of flapped plate, $F_l/(\frac{1}{2}\rho V_A^2 c)$
$C_{l,\alpha}$ = derivative of C_l with respect to α; derivatives with respect to γ, θ, and h are $C_{l,\gamma}$, $C_{l,\theta}$, and $C_{l,h}$, respectively
C_{lf} = flap lift coefficient, $F_{lf}/(\frac{1}{2}\rho V_A^2 c_f)$
C_m = moment coefficient of flapped plate around the $0.37c$ point; C_m-moment$/(\frac{1}{2}\rho V_A^2 c^2)$
$\bar{C}_m$ = moment coefficient of flapped plate around leading edge
$C_{m,\alpha}$ = derivatives of C_m with respect to α; derivatives with respect to γ, θ, and h are named similarly
C_{mf} = flap moment coefficient around the junction, $M_f/(\frac{1}{2}\rho V_A^2 c_f^2)$
c = chord of flapped plate
c_f = flap chord
h = height of quarterchord point of the flapped plate above ground
N = number of nodes between vortex elements
NT = number of time steps
$\boldsymbol{n}_i$ = unit normal vector at control point i
Q_i = normal velocity induced at control point i by a unit circulation at the trailing edge
$\boldsymbol{V}_A$ = linear velocity vector of the flapped plate
V_{AX}, V_{AY} = X and Y components of $\boldsymbol{V}_A$
α = angle of attack
Γ_c = circulation of starting (trailing-edge) vortex
Γ_j = circulation of vortex core at node j
γ = flight-path angle
Δ = change in parameter
δ = flap angle
θ = pitch angle

Introduction

THE longitudinal stability of an aircraft depends on the changes in its aerodynamic coefficients, relative to the changes in parameters describing its motion, such as the angle of attack. The ratios of these changes are described by the derivatives that are known as the stability derivatives.[1] These derivatives are usually determined from the aerodynamic characteristics of the aircraft in a steady uniform flow, free of ground effects. This approach is satisfactory as long as the aircraft is far from ground. However, during landing and takeoff, and for vehicles that intentionally fly near ground, such as "wing in ground," ground effects on the aerodynamic coefficients cannot be ignored. Many studies have shown that these coefficients can increase sharply near ground and, therefore, their derivatives are expected to be different from those obtained far from ground.

The effect of ground on the stability of an aircraft was recognized by Stanfenbiel and Schlichting.[2] In fact, they corrected the condition of static height stability (introduced earlier by others) by including the derivatives: $C_{L\alpha}$, $C_{M\alpha}$, and C_{Mh}, in addition to the already existing C_{Lh}. However, these derivatives were determined from "steady ground effect" analysis. In this analysis, steady flow is solved around the airfoil while at various heights above the ground to simulate the flow as the airfoil approaches ground. This approach has been shown by the authors[3,4] and others[5] to be flawed because of the inherent unsteady nature of the flow as the airfoil approaches ground. These unsteady effects were further shown to produce changes in the aerodynamic coefficients, in many instances, substantially different from those obtained by the steady approach. Nuhait[6] gives an extensive literature review on ground effect in general. Kumar[7] considered some aspects of the stability problem associated with wings in ground effect. He treated the flow as quasisteady and computed the stability derivatives at fixed height above the ground.

The objective of this article is to calculate the stability derivatives of a flapped-plate in ground effect using the unsteady approach. Derivatives with respect to height above ground, flap angle, and flight path, and pitch angles are determined. It is interesting to note that in the commonly used steady approach, the effect of the flight path cannot be accounted for and, therefore, the derivatives with respect to its angle are not calculated. To calculate these important derivatives, one must use unsteady ground effect analysis. This appears to be lacking in the literature. It should be noted that the results presented in this article are not intended for direct application. Rather, they demonstrate the importance of including unsteady effects in evaluating the derivatives.

A flapped plate rather than a single plate was chosen for the present work because of the common use of flaps near ground to avoid the loss of lift due to speed limitations. We feel flaps should be included in any ground effect model;

Received May 23, 1993; revision received March 29, 1994; accepted for publication April 4, 1994.

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something that we also find lacking in published unsteady ground effect models. The unsteady model described earlier by the authors[3] is used in the present calculations after being extended to account for the flap. In doing so we investigated the possibility of allowing vorticity to be shed at the junction between the plate and the flap (hinge), in addition to the usual shedding at the trailing edge.

Unsteady Ground Effect Model

In this article, a plain flap is hinged at the trailing edge of a flat plate. As a result, the model used here is a simple extension of the thin plate model described earlier in detail by the authors.[3] The main features of the model are summarized below.

The flow is treated as two dimensional, incompressible, and inviscid. The method of images is used to represent the ground effect. The images of the plate, flap, and their wakes are placed below the ground plane, thereby making the ground a streamline as shown in Fig. 1. In the figure, two coordinate systems and some definitions are given. The flapped plate and its wake and their images are simulated by vortex sheets. At the start of motion, no wake exists yet. The vortex sheets representing the flapped plate and its image are discretized into a number of elements (each has N nodes and $N - 1$ elements). A uniform vorticity distribution over each element is approximated by an equivalent point vortex at the quarterchord point of the element. The tangency condition is satisfied at one control point on each element located at its middle. The unsteady flow is simulated by placing a vortex core with unknown circulation Γ_c at the trailing edge of the flap.[8] The position of the origin of the moving frame (point A), which is attached to the plate, θ, and δ, must be given as functions of time in order to obtain the solution. γ is proportional to the vertical speed of the plate V_{AY} (with respect to the ground-fixed frame in Fig. 1). They are interrelated through the relation:

$$\gamma = \tan^{-1}(-V_{AY}/V_{AX}) \tag{1}$$

In this article the flapped plate moves along a straight line (i.e., γ = const). The no-penetration boundary condition at a control point i is given by

$$\sum_{j=1}^{N-1} A_{ij}\Gamma_j - Q_i\Gamma_c = \boldsymbol{V}_A \cdot \boldsymbol{n}_i \tag{2}$$

and the principle of conservation of circulation with respect to time (Kelvin's theorem) is expressed as

$$\sum_{j=1}^{N-1} \Gamma_j - \Gamma_c = 0 \tag{3}$$

The solution of a system of equations consisting of Eq. (2) applied at $(N - 1)$ control points in addition to Eq. (3), at the end of the first time step, yields the circulations bound to the flapped plate and the circulation of the core at the flap trailing edge. At the beginning of the second time step this vortex core (starting vortex) is shed and convected downstream at the local fluid particle velocity in order to satisfy the unsteady Kutta condition.[8] The wake is generated as a result of shedding and convecting the starting vortex. Once the wake is generated, the effects of the wake and its image must be included in Eqs. (2) and (3). Thus

$$\sum_{j=1}^{N-1} A_{ij}\Gamma_j - Q_i\Gamma_c = (\boldsymbol{V}_A - \boldsymbol{V}_{\text{wake}}) \cdot \boldsymbol{n}_i \tag{4}$$

$$\sum_{j=1}^{N-1} \Gamma_j - \Gamma_c = \sum_{k=1}^{NT-1} \Gamma_{\text{wake},k} \tag{5}$$

where $\boldsymbol{V}_{\text{wake}}$ is the velocity induced by the vortex cores in the wake and their images at control point i, NT is the number of time steps the solution has advanced, and $\Gamma_{\text{wake},k}$ is the circulation of a vortex core k in the wake. At the end of the second time step, the circulations are computed by solving the system of equations [Eqs. (4) and (5)]. At the beginning of the third time step, the new vortex core at the flap trailing edge is shed and convected to its new position as required by the unsteady Kutta condition. Simultaneously, the starting vortex is convected to a new position in order to make the pressure continuous across the wake. The wake grows as a

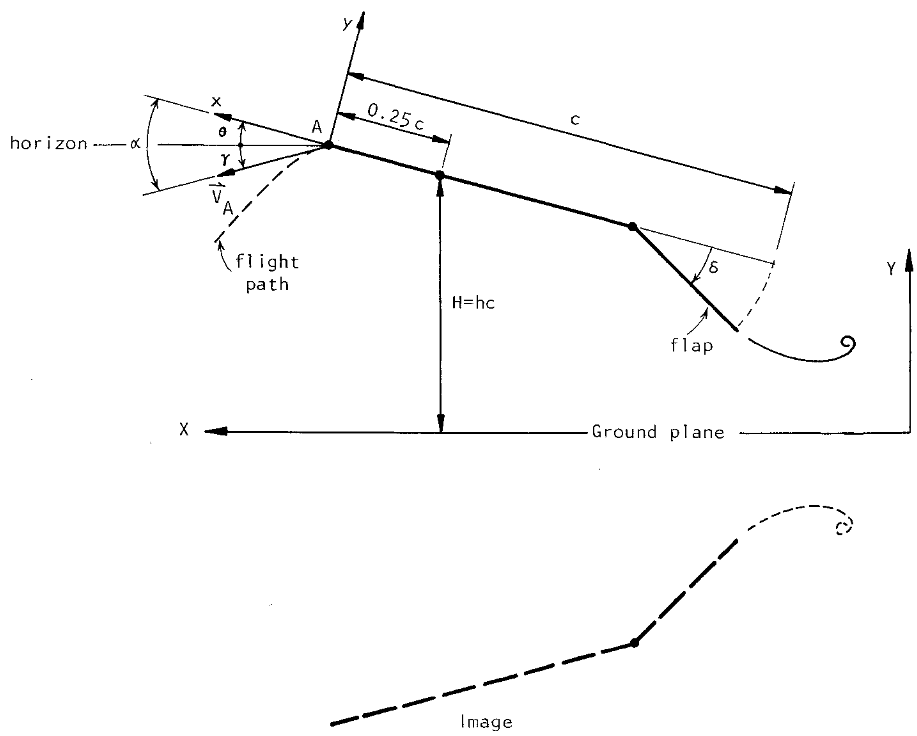

Fig. 1 Definition sketch.

result of shedding and convecting vortices. Thus, the distribution of vorticity in, and shape of, the wake are computed as part of the solution. The solution procedure can be repeated to any desired number of time steps.

The aerodynamic force and moment are computed by integrating the pressure-jump distribution over the flapped plate. The pressure jump across each element is found at the control point of that element by using the unsteady Bernoulli's equation as described in Ref. 3.

In the case when there is a gap between the plate and the flap, the airfoil is treated as two flat plates with vorticity shed from both trailing edges. Equation (4) is then modified to include the effect of the vortex core at the plate trailing edge and the plate wake. Equation (5) is replaced with two equations resulting from the requirement that the circulations around the plate and the flap and their wakes remain constant.

Results and Discussions

First we present some results on extending the model to account for the flap, then we present the results on the stability derivatives obtained by this model and compare them with those obtained by the steady ground effect model. In this article the chord of the flap is taken to be 20% of the total c.

Investigation of the Model

The present model was verified in earlier studies for flat[3] and cambered[4] plates, each moving near ground. We found no results available for a flapped plate in unsteady ground effect to compare with. Because of earlier extensive verifications, the model is expected to work well for a flapped plate, provided that the flap is accounted for properly. To test that, we set the plate to move impulsively from rest very far from ground, and allow the unsteady solution to progress in time until the aerodynamic coefficients reach asymptotic (steady) values while the plate is still outside ground effect. These values are then compared with the results obtained from the closed-form expressions of the thin-airfoil theory.[9] Figure 2 shows such a comparison for a plate with $\delta = 10$ deg at three angles of attack $\alpha = 0$, 5, and 10 deg. These values of α and δ are within the range of validity of the thin airfoil theory. The agreement is excellent for both lift and moment coefficients. Shedding of vorticity was allowed only at the trailing edge of the flap in obtaining these results.

For large values of δ, there is a possibility of vorticity shedding at the junction between the flap and the main plate (even if there is no grap), in addition to the usual shedding at the trailing edge. This possibility is expected to increase if there is a gap between the plate and the flap. This was investigated by allowing shedding of vorticity (a wake) at the junction and comparing the results with those obtained without it. Figure 3 shows such a comparison for $\delta = 10$, 20, and 30 deg at $\alpha = 5$ deg, without considering ground effects. Except at the start of motion (near zero time), the effects of including the wake at the junction on C_l and $\bar{C}_m$ are negligible for $\delta = 10$ deg, and are very small for $\delta = 20$ and 30 deg. Figure 4 shows a similar comparison while including ground effects for a plate with $\delta = 20$ deg, moving at $\alpha = 5$ deg and $\gamma = 20$ deg. The figure is truncated to show C_l and $\bar{C}_m$ only near ground. Again, the effects of including a wake at the junction are negligibly small, especially very close to the ground. Based on these results, and to conserve computer time, we decided not to include the wake at the junction in the remainder of the calculations.

Stability Derivatives

These derivatives are calculated from the results of a number of runs in which the flapped plate starts motion impulsively far enough from ground such that the flow around it reaches steady conditions while it is still outside ground effect. Thus, as the flapped plate moves towards the ground, any changes in the flow (unsteadiness) are brought in by the effect of ground. First results are obtained for a baseline case with $\delta_0 = 20$ deg, $\gamma_0 = 20$ deg, and $\theta_0 = -15$ deg ($\alpha_0 = 5$ deg). To obtain the derivatives with respect to δ, e.g., another run is made with $\delta = \delta_0 + \Delta\delta$, while keeping $\theta = \theta_0$ and $\gamma = \gamma_0$. For a small $\Delta\delta$, the derivative of the lift coefficient is approximated by

$$\frac{\partial C_l}{\partial \delta} \simeq \frac{C_l(h, \theta_0, \gamma_0, \delta_0 + \Delta\delta) - C_l(h, \theta_0, \gamma_0, \delta_0)}{\Delta\delta}$$

The derivatives of other aerodynamic coefficients with respect to $\delta(\partial C_m/\partial\delta, \partial C_{lf}/\partial\delta, \partial C_{mf}/\partial\delta)$ are obtained in a similar fash-

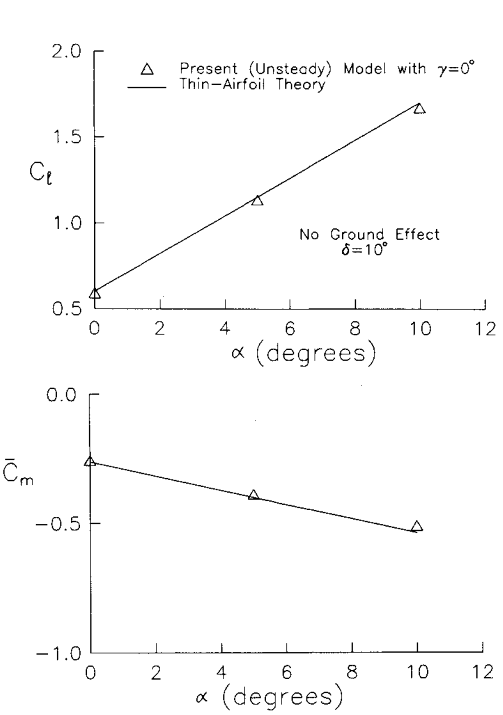

Fig. 2 Steady verification of present model.

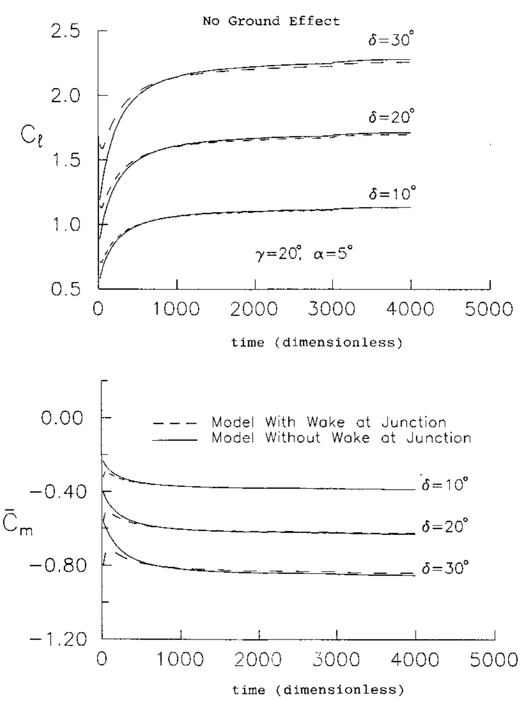

Fig. 3 Effect of wake at the junction of a flapped plate out of ground effect on its aerodynamic coefficients.

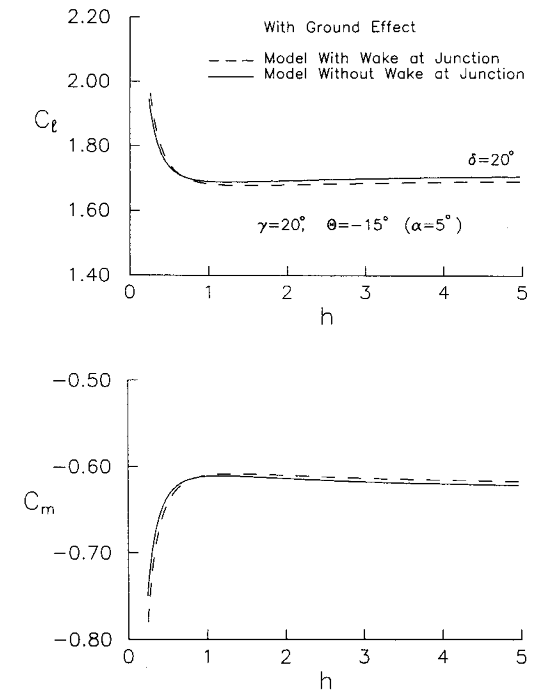

Fig. 4 Effect of wake at the junction of a flapped plate in ground effect on its aerodynamic coefficients.

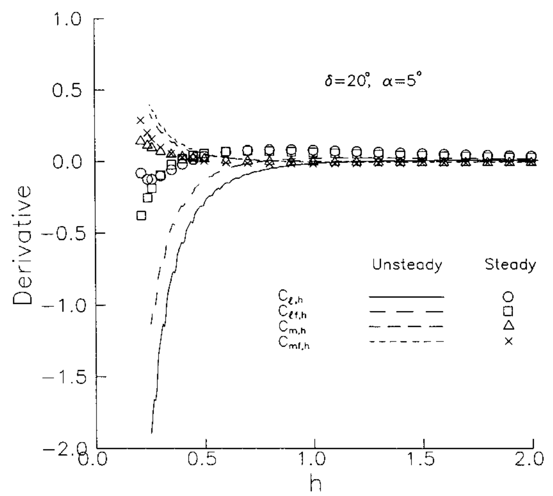

Fig. 5 Derivatives of aerodynamic coefficients with respect to height above ground for flapped plate in ground effect; γ = 20 deg and θ = −15 deg (α = 5 deg) for unsteady analysis, and θ = α = 5 deg for the steady analysis.

ion. The same procedure is used to obtain derivatives with respect to θ and γ. Values of $\Delta\delta$, $\Delta\theta$, and $\Delta\gamma$ used are equal to 0.1 deg. Because of the dependence of the aerodynamic coefficients on the height above ground h as demonstrated by Fig. 4, all the above derivatives are expected to be functions of h. The derivatives of the aerodynamic coefficients with respect to h itself are also important in stability analysis near ground. These are obtained by numerical differentiation of the coefficients (obtained in the base run) with respect to h. The derivative at a point is obtained by fitting a parabola through it and the two neighboring points.

Figure 5 shows the derivatives of C_l, C_m, C_{lf}, and C_{mf}, with respect to h. It is obvious that the derivatives of all coefficients are essentially zeros for $h > 1.5$, indicating insignificant ground effect there. As h drops the derivatives, especially those of C_l and C_{lf} vary considerably. Negative derivatives with respect to h indicate an increase in aerodynamic coefficients towards the ground and vice versa. These results are consistent with those in Fig. 4, which show an increase in C_l and a decrease in C_m as the ground is approached. Note that the moment is taken around the leading edge in Figs. 3 and 4 and around a point at 37% of the chord in the remaining figures. This last location was selected such that zero pitching moment is obtained in steady flight out of ground effect (for α = 5 deg and δ = 20 deg). The derivatives obtained via steady ground effect approach are shown on the figure by symbols. These derivatives agree with those obtained by the unsteady approach for $h > 1.5$, but disagree progressively towards ground. In fact, we observe very substantial differences (order of magnitude) for $h \le 0.5$. The slight oscillations in $(\partial C_l/\partial h)$ observed in Fig. 5 may be attributed to an insufficient number of digits used in writing C_l values from which the derivative is calculated.

Figure 6 shows the derivatives of C_l with respect to δ, γ, and θ. The value of each of these derivatives is roughly constant for $h > 2$, where ground effects are insignificant; $(\partial C_l/\partial\gamma)$ and $(\partial C_l/\partial\theta)$ are approximately equal at a value of around 6 compared to $(\partial C_l/\partial\delta) \simeq 3.2$. As ground is approached, both $(\partial C_l/\partial\theta)$ and $(\partial C_l/\partial\delta)$ drop in a mild fashion, while $(\partial C_l/\partial\gamma)$ initially drops slightly then increases sharply. The agreement between $(\partial C_l/\partial\gamma)$ and $(\partial C_l/\partial\theta)$ outside ground effect and their sharp disagreement close to ground reveal something very important and point out the strong need for unsteady ground effect analysis as explained below.

Since $\alpha = \theta + \gamma$, it follows that

$$\Delta\alpha = \Delta\gamma \quad \text{for} \quad \theta = \theta_0$$

$$\Delta\alpha = \Delta\theta \quad \text{for} \quad \gamma = \gamma_0$$

For a flapped plate with a fixed δ, this leads to

$$\frac{\partial C_l}{\partial\gamma} = \left(\frac{\partial C_l}{\partial\alpha}\right)_{\theta=\theta_0} \quad \text{and} \quad \frac{\partial C_l}{\partial\theta} = \left(\frac{\partial C_l}{\partial\alpha}\right)_{\gamma=\gamma_0}$$

Thus, the results far from ground (Fig. 6) indicate that

$$\left(\frac{\partial C_l}{\partial\alpha}\right)_\theta \simeq \left(\frac{\partial C_l}{\partial\alpha}\right)_\gamma$$

which means that the pitch and flight-path angles do not have any effect on the coefficients and, therefore, on the derivatives there, provided that α is unchanged. In other words whatever

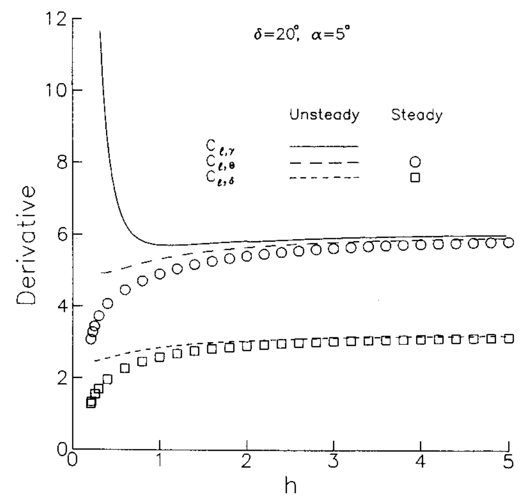

Fig. 6 Derivatives of the lift coefficient of a flapped plate in ground effect with respect to γ, θ, and δ; γ = 20 deg and θ = −15 deg (α = 5 deg) for the unsteady analysis, and θ = α = 5 deg for the steady analysis.

effect θ and γ have is through the effect of a change in α. This is a known fact outside ground effect, and our results just confirm it. However, close to the ground the results (Fig. 6) indicate that

$$\frac{\partial C_l}{\partial \gamma} >> \frac{\partial C_l}{\partial \theta} \quad \text{or} \quad \left(\frac{\partial C_l}{\partial \alpha}\right)_\theta >> \left(\frac{\partial C_l}{\partial \alpha}\right)_\gamma$$

which means that both γ and θ, or γ and α, are important in determining the derivatives. The effect of γ is ignored altogether in the steady ground effect approach, which makes its results not only inaccurate, but also erroneous near ground. The derivatives obtained by the steady approach shown on the figure just confirm this conclusion; they agree with both $(\partial C_l/\partial\theta)$ and $(\partial C_l/\partial\gamma)$ outside ground effects and are considerably lower near ground. Thus, the change in C_l should be expressed, in general, as

$$\mathrm{d}C_l = \frac{\partial C_l}{\partial h}\,\mathrm{d}h + \frac{\partial C_l}{\partial \delta}\,\mathrm{d}\delta + \frac{\partial C_l}{\partial \theta}\,\mathrm{d}\theta + \frac{\partial C_l}{\partial \gamma}\,\mathrm{d}\gamma$$

in which the effect of α is accounted for by θ and γ.

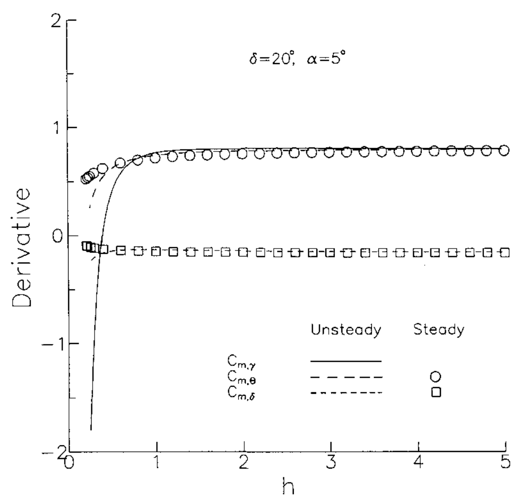

Fig. 7 Derivatives of the moment coefficient of a flapped plate in ground effect with respect to γ, θ, and δ; γ = 20 deg and θ = −15 deg (α = 5 deg) for the unsteady analysis, and $\theta = \alpha$ = 5 deg for the steady analysis.

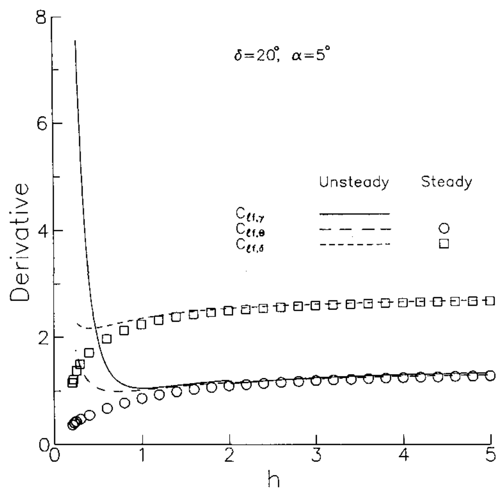

Fig. 8 Derivatives of the flap lift coefficient of a plate in ground effect with respect to γ, θ, and δ; γ = 20 deg and θ = −15 deg (α = 5 deg) for the unsteady analysis, and $\theta = \alpha$ = 5 deg for the steady analysis.

Figure 7 shows the derivatives of the moment coefficient. We observe again that

$$\frac{\partial C_m}{\partial \theta} = \frac{\partial C_m}{\partial \gamma} \simeq \text{const}$$

far from ground. As ground is approached, both derivatives drop slowly initially, then at a sharper rate near ground with $|\partial C_m/\partial\gamma| >> |\partial C_m/\partial\theta|$ very close to the ground. Again, the same reasoning put forward earlier applies to this behavior, enforcing the fact that far from ground changes in α, brought about by changing either γ or θ, have the same effect. Whereas near ground, both γ and θ are important, and derivatives with respect to each of them need to be considered. $(\partial C_m/\partial\theta)$ obtained by the steady analysis agrees with both $(\partial C_m/\partial\theta)$ and

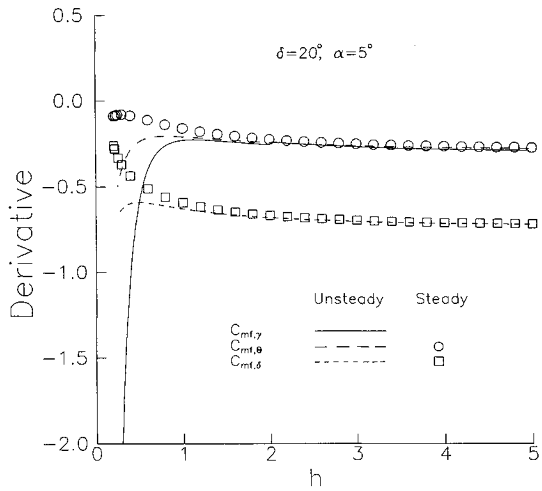

Fig. 9 Derivatives of the flap moment coefficient of a plate in ground effect with respect to γ, θ, and δ; γ = 20 deg and θ = −15 deg (α = 5 deg) for the unsteady analysis, and $\theta = \alpha$ = 5 deg for the steady analysis.

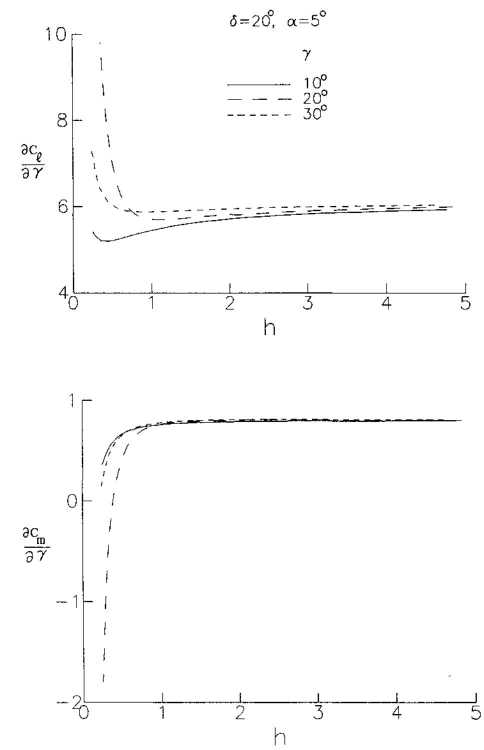

Fig. 10 Effect of flight-path angle on the lift and moment derivatives; δ = 20 deg and α = 5 deg.

$(\partial C_m/\partial\gamma)$ obtained unsteadily far from ground, and disagrees with them close to ground. As for $|\partial C_m/\partial\delta|$, it stays roughly constant until very close to the ground where it drops slightly.

The derivatives of the flap lift coefficient are shown in Fig. 8. The same general observations made when discussing Fig. 6 still apply here, with the exception that $(\partial C_{lf}/\partial\gamma)$ and $\partial C_{lf}/\partial\theta)$ are smaller than $(\partial C_l/\partial\gamma)$ and $(\partial C_l/\partial\theta)$, respectively. Figure 9 shows the corresponding results for the flap moment coefficient. The magnitudes of all the derivatives are negative. This is due to choosing to take the flap moment around the hinge. The behavior of the derivatives with h is generally similar to that observed in Fig. 7. In both Figs. 8 and 9 the derivatives obtained steadily disagree sharply with those computed unsteadily for $h < 1.0$. The importance of the effect of γ is further demonstrated by studying its effect on the derivatives for a given α. Figure 10 shows that the effect of γ on $(\partial C_l/\partial\gamma)$ and $(\partial C_m/\partial\gamma)$ is quite substantial, indicating that the second derivatives with respect to γ are also important. The corresponding results for $(\partial C_{lf}/\partial\gamma)$ and $(\partial C_{mf}/\partial\gamma)$ behave similarly and, therefore, are not presented.

Summary and Concluding Remarks

The stability derivatives of a flapped plate moving in ground proximity were evaluated using an unsteady ground effect model developed previously after extending it to account for the flap. Vorticity is shed as usual from the trailing edge (of the flap) as required by the unsteady Kutta condition when the airfoil approaches the ground. When vorticity is allowed to be shed from the junction point between the plate and the flap, the results were essentially the same. Because of this, and to conserve computer time, the final model used to generate the derivatives did not include shedding (wake) at the junction. Derivatives with respect to pitch angle, flight-path angle, flap angle, and the height above ground were computed at various heights for the nominal values: $\delta_0 = 20$ deg, $\gamma_0 = 20$ deg, and $\theta_0 = -15$ deg. Similar results were obtained using the steady ground effect approach for $\delta_0 = 20$ deg and $\alpha_0 = 5$ deg.

Both the results of the steady and unsteady models showed substantial effect for ground on the derivatives, indicating that they should never be computed from aerodynamic data obtained far from ground when the airfoil is moving in ground proximity. Moreover, the unsteady derivatives were different from the steady ones close to the ground, again indicating that the steady approximation is not accurate. In particular, the results showed that the effects of γ on the aerodynamic coefficients and their derivatives are extremely important near ground. Therefore, derivatives with respect to γ, in addition to those with respect to θ (or α), must be considered. The steady approach ignores, by its own nature, the effect of γ altogether. The results presented in this article are not intended for direct application. Rather, they demonstrate the importance of including the unsteady effects in evaluating the derivatives near the ground.

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